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Three-dimensional numerical study of supercritical pressure effect on heat transfer of cryogenic methane

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Abstract: A three-dimensional numerical study of the turbulent convective heat transfer of the cryogenic methane flowing inside a square engine cooling channel under supercritical pressures was systematically conducted. Numerical results indicate that increasing the fluid pressure results in enhanced heat transfer of the cryogenic methane under supercritical pressures. At the pseudo-critical temperature under a corresponding supercritical pressure, drastic property variations cause heat transfer deterioration and sharp wall temperature increase at a high wall heat flux of 7 MW/m^2 . A modified Jackson and Hall heat transfer equation, which can be used for supercritical heat transfer calculations of the cryogenic methane, has been successfully established in this paper.

Key words: supercritical heat transfer; supercritical pressure; regenerative cooling; cryogenic methane; numerical study

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Nomenclature

C	Constant in turbulent model	p	Pressure, Pa
c_p	Constant-pressure heat capacity, $\text{J}/(\text{kg} \cdot \text{K})$	Pr	Prandtl number
e_i	Total internal energy, J/kg	q	Heat flux, W/m^2
G_k	Turbulent generation term	Re	Reynolds number
H	Enthalpy, J/kg	T	Temperature, K
k	Turbulent kinetic energy, J/kg	u	Velocity, m/s
Nu	Nusselt number		

Greek symbols

ϵ	Turbulent dissipation rate, m^2/s^3	ρ	Density, kg/m^3
λ	Thermal conductivity, $\text{W}/(\text{m} \cdot \text{K})$	σ	Turbulent Prandtl number
μ	Viscosity, $\text{N} \cdot \text{s}/\text{m}^2$	τ	Viscous stress tensor, N/m^2

Subscripts

b	Bulk parameter	w	Wall
c	Critical value	t	Turbulent value
k	Turbulent kinetic energy	ϵ	Turbulent dissipation rate
pc	Pseudo-critical value		

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Introduction

Presently there are significant interests in liquid oxygen (LOX)/Methane liquid-propellant rocket engine development, owing to the advantages of the cryogenic-propellant methane comparing with the other traditional liquid propellants, i. e. its higher density, higher vaporization temperature, less-challenging storage requirements comparing with the cryogenic hydrogen, and its higher specific impulse, superior cooling capability, higher coking limit comparing with kerosene^[1-2]. In a LOX/methane liquid-propellant rocket engine, methane also acts as a coolant for the regenerative engine cooling process. Since the working pressure inside the combustion chamber is generally higher than the thermodynamic critical pressure of methane, the heat transfer process of methane flowing inside the engine cooling channel prior to its injection would thus be under a supercritical pressure.

In this paper, the convective heat transfer under a supercritical pressure with both a sub- and super-critical fluid temperature is defined as the supercritical heat transfer. Therefore, during a supercritical heat transfer process, methane experiences no phase change, but could transit from a liquid-like to a gas-like state with drastic property variations within a narrow temperature range. The supercritical heat transfer process of the cryogenic-propellant methane is influenced strongly by thermophysical property variations and differs significantly from its subcritical counterpart.

Since the supercritical heat transfer phenomenon plays a crucial role in the regenerative engine cooling technology development, it has attracted significant research interests. A number of experimental studies have been carried out for various kerosene fuels. Hitch and Karpuk^[3] conducted experimental studies on the supercritical fluid flow and heat transfer of JP-7 in a vertical tube. They observed significant pressure and temperature oscillations, along with declined local heat transfer coefficients, when the reduced

pressures (p/p_c) was below 1.5 and the tube wall temperature above the corresponding pseudo-critical temperature. Chen and Dang^[4] also performed experimental investigations on the supercritical heat transfer of JP-7, focusing on the heat transfer and coking properties. Zhong et al.^[5] experimentally studied the heat transfer characteristics of China No. 3 kerosene under supercritical conditions, and found drastic changes in the heat transfer characteristics as the hydrocarbon fuel approached its critical state.

In the open literature, however, very few studies of the supercritical heat transfer of the cryogenic methane have been reported. Pizzarelli et al.^[6] made a three-dimensional (3-D) numerical study on the behavior of the cryogenic methane inside the rocket engine cooling channel under supercritical pressures. They investigated the channel aspect ratio effect on the cooling performance of the cryogenic methane, and found that property variations could strongly influence the flow field and heat transfer process. However, supercritical heat transfer coefficient variations (i. e. Nusselt number) were not analyzed in this work. Given its crucially important role in the LOX/methane rocket engine development, the supercritical heat transfer process of the cryogenic methane requires further fundamental investigations. In particular, a proper heat transfer expression should be established for deeper physical understandings of the supercritical heat transfer process and for facilitating engine cooling system design.

In this paper, a numerical study of the supercritical heat transfer of cryogenic methane flowing inside a square engine cooling channel has been systematically conducted. A complete set of conservation equations of mass, momentum, and energy have been solved with accurate calculations of the thermophysical properties. The present numerical investigation focuses on the effect of the channel pressure on the turbulent convective heat transfer of the cryogenic methane. The Nusselt number variation has been examined in detail, and a proper empirical

heat transfer formula has been obtained for supercritical heat transfer analysis.

1 Theoretical formulation

The present numerical study concerns the cryogenic methane flowing inside a square engine cooling channel under supercritical pressures, as shown in Fig. 1. The square channel is 2 mm in width and 600 mm in length. The channel is divided into three sections. The beginning section with 150 mm in length is unheated and used to allow flow field development, the middle section of 300 mm in length is heated with a constant wall heat flux enforced on the top channel surface, and the ending section with 150 mm in length is included to avoid the boundary condition effect on the accuracy of heat transfer calculations. The present numerical study focuses on the forced convective heat transfer process and neglects the buoyancy effect. Because of the geometric and physical symmetry in the present configuration, only half of the channel in the positive z direction is included in the numerical calculations.

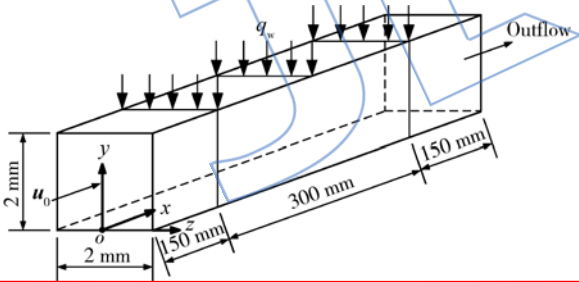


Fig. 1 Schematic configuration

The following conservation equations of mass, momentum, and energy have been numerically solved in the present 3-D study:

$$\nabla \cdot (\rho \mathbf{u}) = 0 \quad (1)$$

$$\nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p - \nabla \cdot \boldsymbol{\tau} \quad (2)$$

$$\nabla \cdot (\rho \mathbf{u} e_t) = \nabla \cdot (\lambda \nabla T) - \nabla \cdot (p \mathbf{u}) \quad (3)$$

The standard k - ϵ turbulent model with an enhanced wall treatment, solving a one-equation model in the near wall region if the meshes are within the viscous sublayer or using a wall function if the meshes are not, is applied in the present numerical study. By putting some numerical

meshes into the viscous sublayer, this approach could avoid the effect of the uncertain wall function on supercritical heat transfer calculations at the heated channel surface. The conservation equations of the standard k - ϵ turbulent model are

$$\nabla \cdot (\rho \mathbf{u} k) = \nabla \cdot \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \nabla k \right] - \rho \epsilon + G_k \quad (4)$$

$$\nabla \cdot (\rho \mathbf{u} \epsilon) = \nabla \cdot \left[\left(\mu + \frac{\mu_t}{\sigma_\epsilon} \right) \nabla \epsilon \right] + C_1 \frac{\epsilon}{k} G_k - C_2 \rho \frac{\epsilon^2}{k} \quad (5)$$

More details concerning these equations and the related parameters can be found in Ref. [7].

A key component for numerically studying the supercritical heat transfer phenomenon of the cryogenic methane is to accurately obtain its thermophysical properties, since these properties would experience strong variations and exert significant effects on the flow field and heat transfer process under supercritical pressures. In order to calculate these properties accurately, the fundamental thermodynamic theories and a Soave-Redlich-Kwong (SRK) equation of state are combined to calculate the thermodynamic properties of methane, i. e. its heat capacity, a Benedict-Webb-Rubin (BWR) equation of state for propane is employed to calculate its density, and the extended corresponding-state method is further applied for evaluating the density and transport properties of methane, i. e. the viscosity and thermal conductivity. More details concerning property calculations can be found in Refs. [8-9]. Discussion of the accuracy of these methods has been provided in Refs. [10-12]. The calculated methane density and constant-pressure heat capacity under various pressures and temperatures are presented in Fig. 2, in which comparisons have also been made against the National Institute of Standards and Technology (NIST) data^[13], showing excellent agreement.

The present numerical model for the supercritical heat transfer of the cryogenic methane has been built into a commercial computational fluid dynamics (CFD) package, Fluent, through

its user coding capability. A pressure-based incompressible numerical algorithm has been applied for the present numerical study, along with the standard inlet, outlet, and wall boundary conditions.

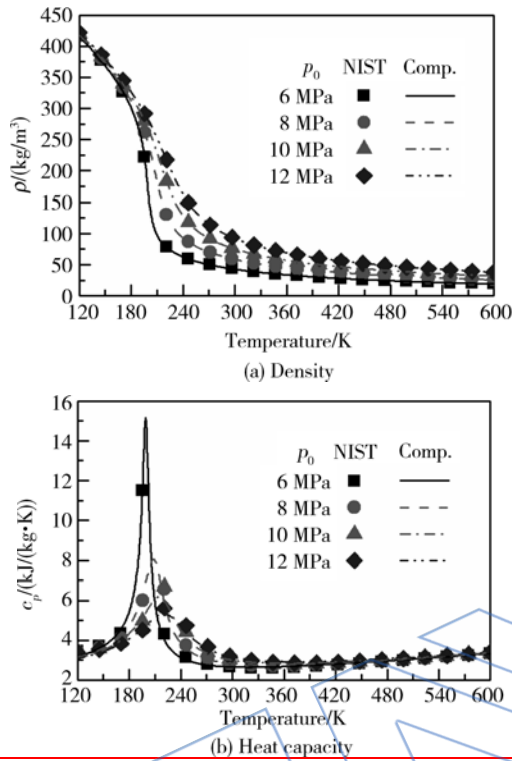


Fig. 2 Thermophysical properties

2 Results and discussion

Prior to extensive numerical studies, a model validation has first been conducted. Since no experimental result exists for the supercritical heat transfer of the cryogenic methane, the present numerical model is validated against experimental data for the supercritical heat transfer of carbon dioxide^[14]. The same computer code, with different input data for calculating the thermophysical properties of supercritical CO_2 , has been used to simulate the exact experimental configurations in Ref. [14]. Comparisons are presented in Fig. 3, showing very good agreement with relative errors within 10% between the two sets of data (P01~P04 are numbers of experimental tests).

A careful grid-independence study has also been conducted. A grid system of $2000 \times 90 \times 45$ in the x , y , and z directions, respectively, is

found sufficient to produce reliable numerical results. Since the standard $k-\epsilon$ turbulent model with an enhanced wall treatment has been applied, the first three layers of grid points close to the heated channel surface are placed into the viscous sublayer (satisfying $y^+ \leq 5$) to avoid the effect of an uncertain wall function on the heat transfer calculations under supercritical pressures.

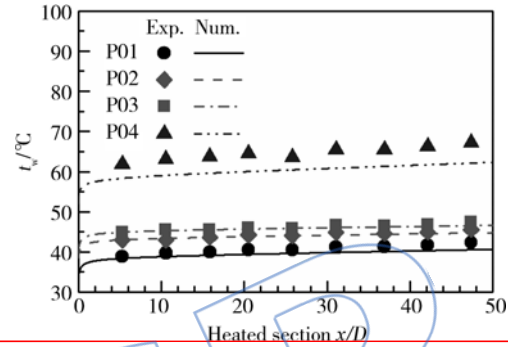


Fig. 3 Model validation against experimental data in Ref. [14]

After the model validation and grid-independence study, extensive numerical investigations of the turbulent convective heat transfer of the cryogenic methane flowing inside a square engine cooling channel under supercritical pressures have been conducted, focusing on the effect of channel pressure. For all the following calculations, the inlet fluid temperature and velocity are set at 120 K and 15 m/s, respectively.

Fig. 4 illustrates the variations of the averaged wall temperature at three wall heat fluxes, ranging from 3 MW/m^2 to 7 MW/m^2 , and under two supercritical pressures of 8 MPa and 12 MPa, respectively. The hydraulic diameter of this square

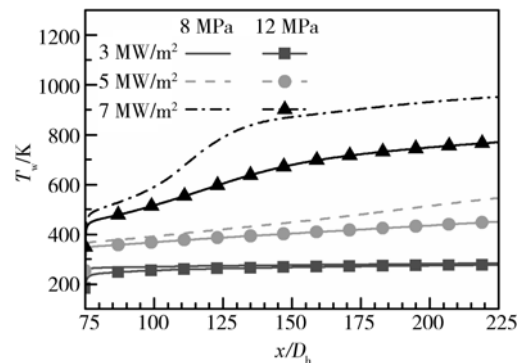


Fig. 4 Averaged wall temperature variations under 8 MPa and 12 MPa

cooling channel, D_h , is 2 mm. Therefore, $x/D_h=75$ is at the beginning of the heated section, while $x/D_h=225$ is at the end. Results in Fig. 4 clearly indicate that increasing the fluid pressure would decrease the wall temperature. This phenomenon is more profound at a higher wall heat flux. For example, in Fig. 4 under 8 MPa, the wall temperature at 5 MW/m^2 is above 500 K, but it decreases to around 400 K under 12 MPa. Once the wall heat flux increases to 7 MW/m^2 , the wall temperature difference under the two supercritical pressures grows even larger.

Fig. 5 provides the wall temperature variations at different z positions at a wall heat flux of 7 MW/m^2 under the two supercritical pressures. The wall temperature exhibits significant increase close to the channel wall ($z=1.0 \text{ mm}$) because of extremely low flow velocity and consequently weak convection heat transfer in this region. Under 8 MPa in Fig. 5, the wall temperature at $z=0 \text{ mm}$ in the middle of the channel and $z=0.4 \text{ mm}$ experiences sharp increase with x/D_h ranging from 105 to 140. This interesting phenomenon is caused by thermophysical property variations at the pseudo-critical temperature. A pseudo-critical temperature is defined as the temperature at which the constant-pressure heat capacity reaches a maximum value under a supercritical pressure. At 8 MPa, the pseudo-critical temperature of methane is around 208 K. As shown in Fig. 5, methane temperature reaches the pseudo-critical temperature approximately at $x/D_h=105$. Afterwards, the volumetric heat capacity of methane experiences an abrupt decrease, as shown in Fig. 6, leading to the sharp increase of the wall temperature. This phenomenon of heat transfer deterioration and wall temperature increase also occur at 12 MPa, but it becomes much weaker since the thermophysical property variations becomes smoother in this case, as shown in Fig. 6.

As shown in Fig. 6, once the thermophysical property variation becomes stable, the wall temperature variation in Fig. 5 gets into a smooth

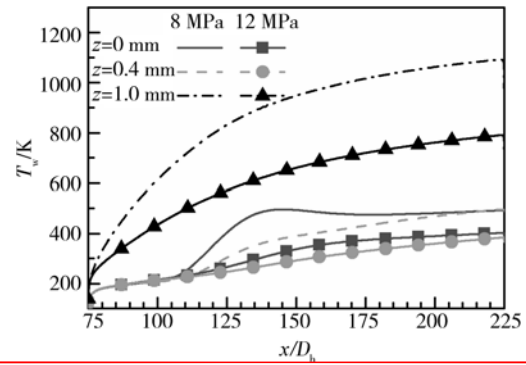


Fig. 5 Wall temperature variations at different z positions under 7 MW/m^2

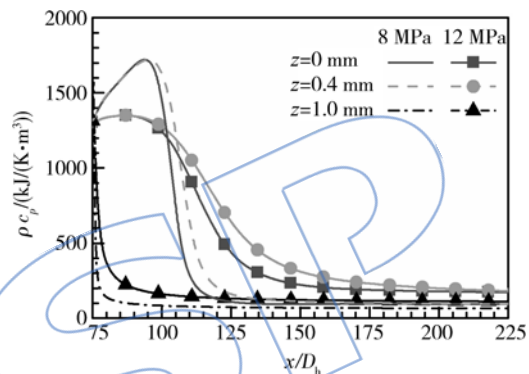


Fig. 6 Volumetric heat capacity variations at different z positions under 7 MW/m^2

state and could even slightly decrease owing to large fluid velocity increase. The fluid velocity increases along the flow direction because the methane density decreases with rising fluid temperature. The 3-D temperature and velocity distributions in different cross sections at 7 MW/m^2 and 8 MPa are illuminated in Fig. 7.

In order to find a proper heat transfer expression for the supercritical methane, we have calculated the Nusselt number based on the present numerical results and made comparisons with that from a slightly modified Dittus-Boelter equation, as shown in Fig. 8(a). Comparing with the original Dittus-Boelter equation^[15], its constant is changed from 0.023 to 0.016 in the present version for heat transfer calculations in a square channel instead of a tube. The modified Dittus-Boelter equation is in the following form:

$$Nu_b = 0.016 Re_b^{0.8} Pr_b^{0.4} \quad (6)$$

where the Nusselt number is defined as

$$Nu_b = \frac{h D_h}{\lambda_b} \quad (7)$$

As shown in Fig. 8 (a), since the results from the modified Dittus-Boelter equation varies in an opposite trend from the numerical results as the wall heat flux increases. The parameter in Eq. (6) is adjusted in a way that an optimum fit between the empirical and numerical results can be obtained at a low wall heat flux of 3 MW/m^2 under both supercritical pressures. In Fig. 8(a), the difference between the two sets of data significantly grows as the wall heat flux increases. Therefore, the modified Dittus-Boelter equation is inapplicable for supercritical heat transfer calculations of methane at a high wall heat flux, i. e. 7 MW/m^2 .

In our previous numerical studies, a slightly

modified Jackson and Hall^[16] heat transfer equation worked well for supercritical heat transfer calculations of hydrocarbon fuels inside a mini tube^[11], because of its accommodation of strong property variations. In this paper, the Jackson and Hall equation, originally proposed for supercritical carbon dioxide and water, is further modified for supercritical heat transfer of the cryogenic methane in a square cooling channel. The modified Jackson and Hall equation can be expressed as

$$Nu_b = 0.015 Re_b^{0.82} Pr_b^{0.5} \left(\frac{\rho_w}{\rho_b} \right)^{0.3} \left(\frac{\bar{c}_p}{c_{pb}} \right)^n \quad (8)$$

where the parameters, $\bar{c}_p$ and n , are defined as

$$\bar{c}_p = \frac{H_w - H_b}{T_w - T_b} \quad (9)$$

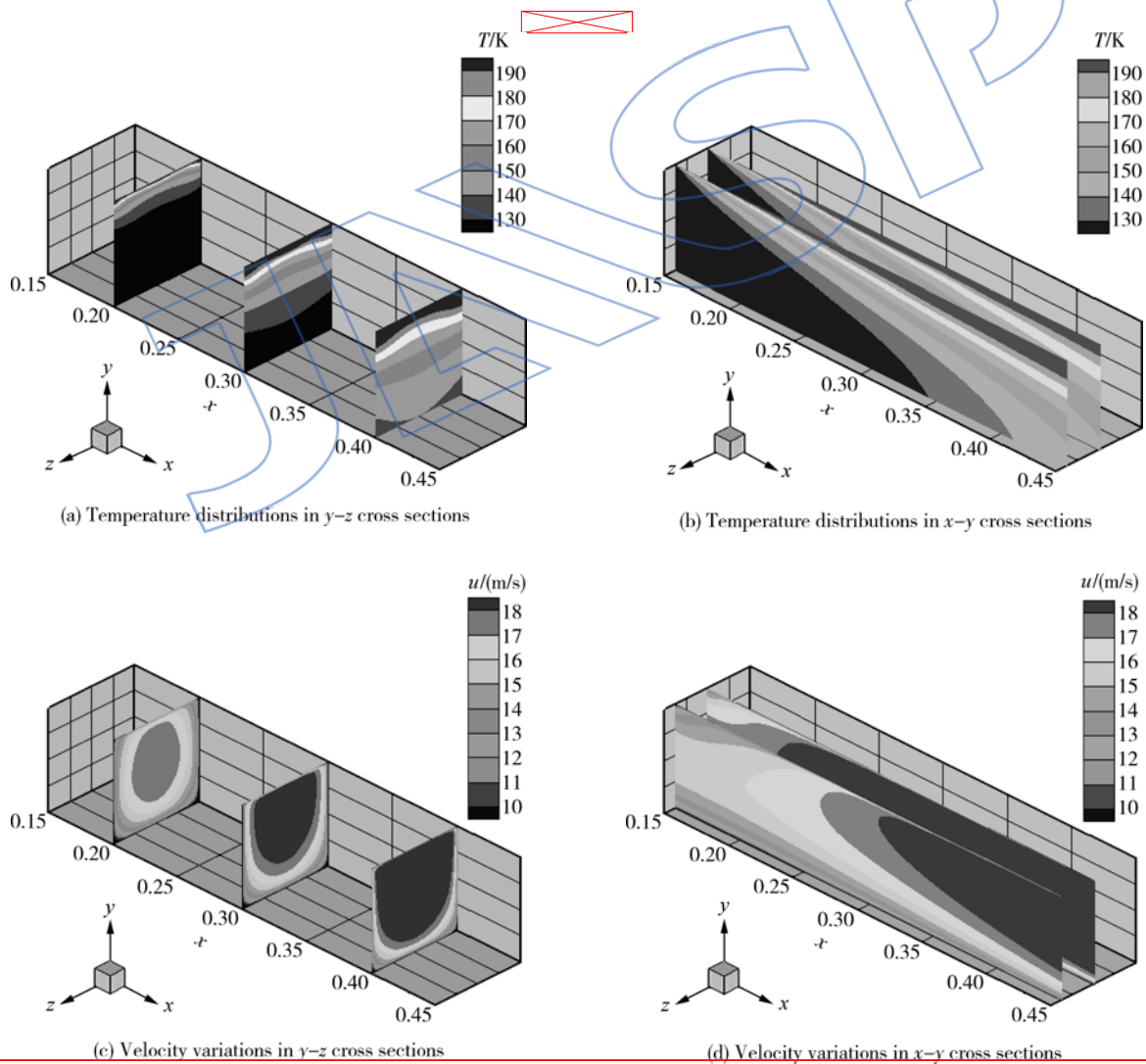


Fig. 7 Temperature distributions and velocity variations in different cross sections under 7 MW/m^2 and 8 MPa

$$n = \begin{cases} 0.4 \\ 0.4 + 0.2 \left(\frac{T_w}{T_{pc}} - 1 \right) \\ 0.4 + 0.2 \left(\frac{T_w}{T_{pc}} - 1 \right) \left[1 - 5 \left(\frac{T_b}{T_{pc}} - 1 \right) \right] \end{cases} \quad \begin{aligned} &T_b < T_w \leq T_{pc} \quad \text{or} \quad 1.2 T_{pc} \leq T_b < T_w \\ &T_b \leq T_{pc} < T_w \\ &T_{pc} < T_b \leq 1.2 T_{pc} \quad \text{and} \quad T_b < T_w \end{aligned} \quad (10)$$

As shown in Fig. 8(b), results from the numerical calculations and the empirical equation vary in the same trend as the wall heat flux increases. Neglecting the thermal entry region, the relative errors between the two sets of data are within 25%, very good for an empirical heat transfer equation.

Comparisons of the numerical results and those from Eq. (8) under 12 MPa are further illustrated in Fig. 9. The modified Jackson and Hall equation also works well for all three cases. The relative errors between the two sets of data are mostly within 20%. Therefore, the modified Jackson and Hall heat transfer equation can be

used for supercritical heat transfer calculations of the cryogenic-propellant methane inside a square engine cooling channel.

Although the present numerical studies focus on fundamental aspect of the supercritical heat transfer of the cryogenic methane under a constant wall heat flux, the heat transfer formula established in this paper could be used for facilitating engine cooling system design and for analyzing engine performance coupled with the regenerative cooling process. For example, the heat transfer formula could be used for boundary condition treatment in a detailed engine combustion calculation.

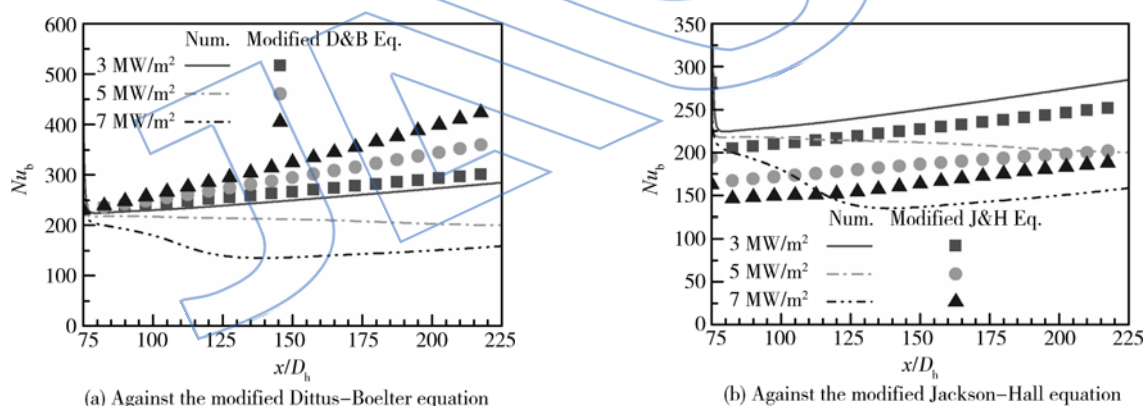


Fig. 8 Comparisons of Nusselt number variations under 8 MPa

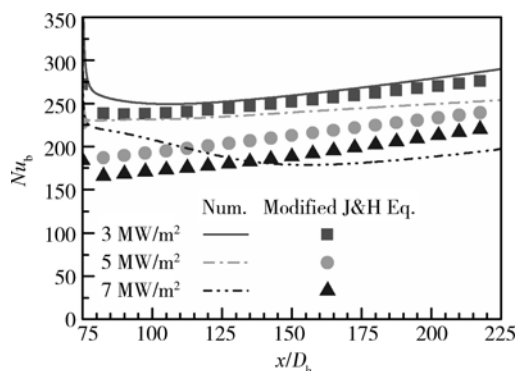


Fig. 9 Comparisons of Nusselt number variations against the modified Jackson-Hall equation under 12 MPa

3 Conclusions

In this paper, a 3-D numerical study of the turbulent convective heat transfer of the cryogenic-propellant methane flowing inside a square engine cooling channel under supercritical pressures has been systematically conducted. A complete set of conservation equations of mass, momentum, and energy has been numerically solved with accurate calculations of the thermo-physical properties. Numerical results illuminate that increasing the fluid pressure would result in

enhanced heat transfer under supercritical pressures of methane. At the pseudo-critical temperature under a corresponding supercritical pressure, drastic property variations could lead to heat transfer deterioration and sharp wall temperature increase at a high wall heat flux, i. e. 7 MW/m^2 . This phenomenon is more profound for methane under 8 MPa than that under 12 MPa, indicating again that increasing the fluid pressure would benefit supercritical heat transfer. A modified Jackson and Hall heat transfer equation, which can be used for supercritical heat transfer calculations of the cryogenic methane inside a square engine cooling channel, has been successfully established in the present study.

Acknowledgments

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